Optimal Placement of Sensors for Trilateration: Regular Lattices vs Meta-heuristic Solutions

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Abstract. Location-aware applications, such as indoor robot navigation or human activity monitoring, require the location estimation of moving elements, by using ultrasonic, infrared or radio signals received from sensors deployed in the workplace. These sensors are commonly arranged in regular lattices on the ceiling. However, this configuration is not optimal for location estimation using trilateration techniques, in terms of positioning precision, maximum coverage and minimum singular cases. This paper shows how non-regular optimal sensor deployments, generated with a new meta-heuristic optimization methodology (Diversified Local Search - DLS), outperforms regular lattices for trilateration.

Keywords: Heuristic optimization, Diversified Local Search, Trilateration singularities, Optimal sensor locations.

1 Introduction

Location-aware systems find important applications in a diverse range of fields, such as outdoors vehicle guidance, reliable robot navigation or human activity monitoring. The most successful outdoor location solution is the well known Global Positioning System (GPS). However, there is not an equivalent solution for indoors. Some solutions developed for indoor use rely on ultrasonic, infrared or radio technology [1, 2], but a definitive system has not been accepted yet, due to several technical reasons and cost constrains. Indoor Local Positioning Systems (LPS) is an important and open research field.

Most location systems use trilateration techniques for position estimation [3–5]. They estimate the unknown coordinates of a mobile, given the known coordinates of a group of reference points (sensors, antennas or satellites), and the distances between each sensor and the mobile. The number of sensors and available distances has to be sufficient in order to find the unknowns by solving a nonlinear system of equations.

For proper trilateration, the sensor distribution in relation to the mobile location has to be favorable. Unfavorable deployments can induce singularities and

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low precision in the position estimations. Therefore, it is important to know the right placement of sensors, in order to avoid bad configurations. The well known metric termed Dilution Of Precision (DOP), can be used to detect such configurations. The DOP is a dimensionless number, that decreases when the trilateration problem becomes better conditioned (good geometry among sensors). It can be expressed as the ratio of standard deviations between location estimations and measured distances [6, 7], given as:

$$DOP = \sqrt{\sigma_{x_k}^2 + \sigma_{y_k}^2 + \sigma_{z_k}^2} / \sigma_{d_k}. \tag{1}$$

There are two main trilateration techniques called spherical (ST) and hyperbolic (HT). ST uses absolute distances between sensors and mobile device, whereas HT uses differences of distances between sensors and mobile with respect to one reference sensor. Due to this fact, ST requires one sensor less than HT. In ST systems the singularities are generated when the available sensors are positioned in a straight line, whilst in HT systems it also happens when the mobile location is such that differences of distances becomes null. Therefore, according to the selected technique, it is possible to find different DOP values, singular areas (SA) and non-coverage areas (NC) which are regions where the available amount of sensors is not enough to solve the equation system. A representative case of singularity using ST is shown in figure 1. Figure 1a illustrates how a mobile moves from a non-singular place (A) to a singular place (B) where three sensors are aligned. Figure 1b shows an overhead view of figure 1a with an indication of DOP values, SA and NC areas. Figure 2 shows a case of singularity using HT. Notice that the same sensor distribution is used. However, DOP, NC and SA in HT are worse in comparison with the ST case (see figure 2b).

When the work area has obstacles, walls or an irregular shape, finding an optimal sensor deployment is a non-trivial task. It is possible to consider a regular distribution using a large amount of sensors in order to avoid SA [8]. However, this solution is not recommendable since it increases the cost of the system. Several 2D and 3D location system based on a trilateration technique have been developed. Some of these systems used regular sensor deployments [5, 9], but do not consider singularities or the low precision caused by aligned or coplanar sensors. As a result, these systems require a redundant amount of sensors. Another works have studied optimal sensor deployment for 2D and 3D scenarios [10, 11]. The optimal 2D solution consisted of three sensors forming an equilateral triangle and a fourth one at the center of the triangle; both sensors and mobile were on the same plane. 3D optimal solutions were based on sensors distributed on a unit spherical surface, the sensors were located on the vertices of 3D shapes called Platonic solids (tetrahedron, octahedron, etc). However, all these sensors distributions are not valid for a realistic deployment in indoor environments since do not have into account blockage of signals, free space installation, etc.

This work aims at demonstrating how optimal sensor deployments, generated with a meta-heuristic optimization strategy called Diversified Local Search (DLS), outperform regular arrangements, such as square or triangular lattices. DLS searches for optimal solutions providing maximum positioning accuracy

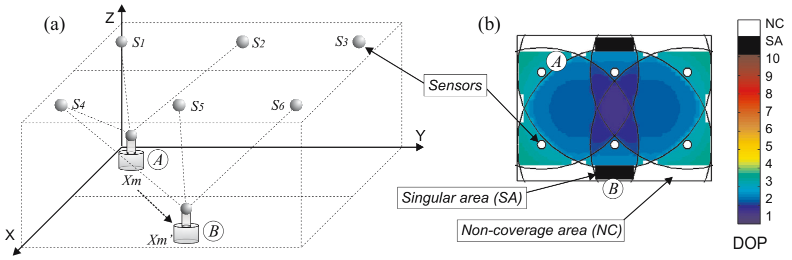

Fig. 1. A representative case of singularities in a ST system using a regular square lattice sensor deployment: (a) The mobile device moves from a place (A) without singularities to a singular place (B). (b) Top view with DOP values, NC and SA areas.

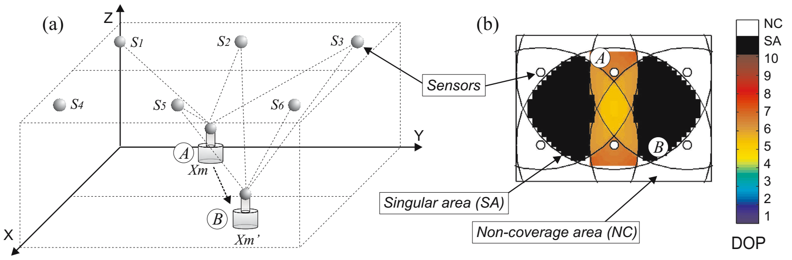

Fig. 2. A representative case of singularities in a HT system using a regular square lattice sensor deployment: (a) The mobile device moves from a place (A) without singularities to a singular place (B). (b) Top view with DOP values, NC and SA areas.

(low DOP) and maximum coverage with a few number of sensors (low cost). Realistic scenarios are considered; therefore sensors were on the ceiling in order to achieve Line-Of-Sight (LOS) of signals. Next section describes the methodology used to calculate and generate the best sensor deployments. In section 3, results are presented, showing examples of optimal deployment solutions applying ST and HT. Finally, a discussion and conclusions are given.

2 Methodology

2.1 Quality of Sensor Deployments

Two factors to describe the performance of sensor deployments were considered: **Precision** and **Non-Availability**. These factors were combined according to the following fitness function:

$$f(\Omega) = \underbrace{\overline{\mathrm{DOP}} \times \mathrm{K1}}_{Precision} + \underbrace{\mathrm{NAR} \times \mathrm{K2}}_{Non-Availability} \ . \tag{2}$$

Where:

- Ω: It is given by the set $\{[x_i, y_i, z_i] : i = 1, ..., n\}$ of coordinates of all sensors included in the solution.

  $\overline{\textbf{DOP}}$: It is the mean of DOP values over the desired localization area.
- **NAR**: It is the Non-Availability Ratio. It is given as: (NC+SA)/*Area*. Where, *Area* is the location area that we aim to cover with the positioning system.
- **K1 and K2**: These are weights to balance the level of importance desired for each term in the fitness function.

The fitness function (2) aims at maximizing precision and the available area for location of mobile device. Therefore, DOP and NAR must be reduced in order to find optimal sensor deployments. To perform the DOP, NC, SA and Area computations, the intersections in a square grid with 10 cm of resolution, over the desired localization area, were used as the evaluation points.

2.2 Methods to Find the Best Sensor Deployments

In equation (2), values for K1 and K2 were selected to get deployments solutions according to desired trade-off between precision and availability. These were K1=15 and K2=400 for both ST and HT cases. Using these weights, the best regular sensors deployment based on square and triangular lattices were found by changing uniformly inter-sensor distances (approx. from 0.5 to 2.5m). These solutions corresponded to regular deployments that produced the minimum value in (2). For meta-heuristic optimization, we start with an grouped initial distribution (very close sensors), the equation (2) was minimized as a multi-objective problem. To minimize it, the DLS optimization method [13] was used. It is based on a combination of Local and Tabu search. The aim of this search is to find the best DLS sensor deployments. Following this, a 2D cost function space between DOP and NAR was used to compare the performance of regular and DLS solutions. The cost of the positioning system is important and it is proportional to the number of sensors used. Therefore, a low number of sensors was kept constant during the search of best deployments. It allowed to observe the performance of regular and DLS deployments using few sensors.

Using the DLS method, different values for K1 and K2 yield to different optimal solutions in terms of DOP and NAR. In order to find the best possible solution for different K1 and K2 values, the Pareto Optimality Criterion (POC) was applied, where different factors are balanced in such a way that no improvement can be achieved by decreasing one factor without increasing another. Applying POC, a curve of solutions called *Pareto Frontier* was found, by minimizing the equation (2) using DLS method, changing the values of K1 and K2, in each search, from K1$\gg$K2 (the best DOP and the worst NAR) and vice versa.

3 Results

Experiments were defined for a square area (of 25 m^2) in order to consider a room with constant LOS propagation between the sensors and the mobile device, and

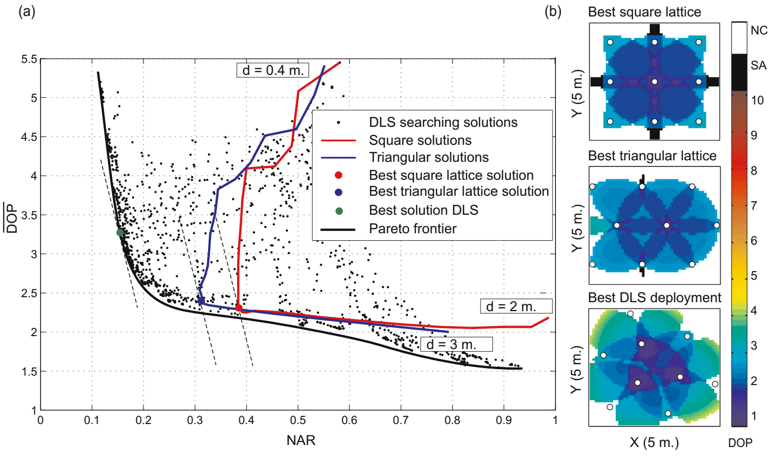

Fig. 3. Optimization results for a square area applying ST: (a) Cost function space, (b) Best sensor deployments

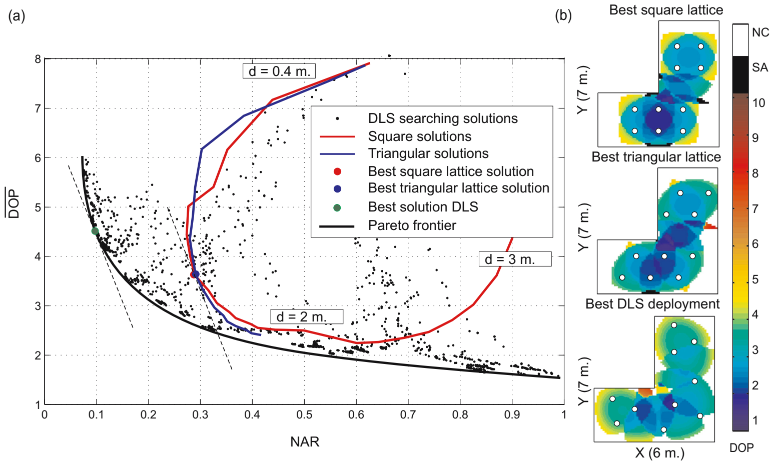

Fig. 4. Optimization results for a L-shaped area applying ST: (a) Cost function space, (b) Best sensor deployments

a L-shaped area (of 30 m^2) whose outer-corner walls causes blockage of signals. A realistic scenario with 2 m height was simulated. The sensor locations were on ceiling and the location area was on floor. The number of sensors was 9 and 14 for the square and L-shaped areas, respectively, in order to observe how deployments with few sensors can be improved. Considering a sensibility limited by the reception pattern of sensors at 45^{o}, the link between the sensor and the mobile was available in a circumference of radius 2 m over the location area just below each sensor. First, it is shown a comparison among regular lattice and DLS solutions, using ST and then using the HT.

Table 1. Values of f(Ω), DOP and NAR for all the best sensor deployments found

	Spherical Tecnique (ST)						Hyperbolic Technique (HT)					
	Square room			L-shaped room			Square room			L-shaped room		
	f(Ω)	DOP	NAR	f(Ω)	DOP	NAR	f(Ω)	DOP	NAR	f(Ω)	DOP	NAR
Best square lattice	188.7	2.298	0.385	170.8	3.619	0.291	183.9	6.714	0.208	301.0	7.145	0.484
Best triangular lattice	159.2	2.405	0.308	169.8	3.626	0.288	195.8	6.570	0.243	309.2	7.248	0.501
Best DLS deployments	**110.4**	3.183	0.156	**108.8**	4.486	0.104	**157.4**	7.127	0.126	**254.8**	7.328	0.362

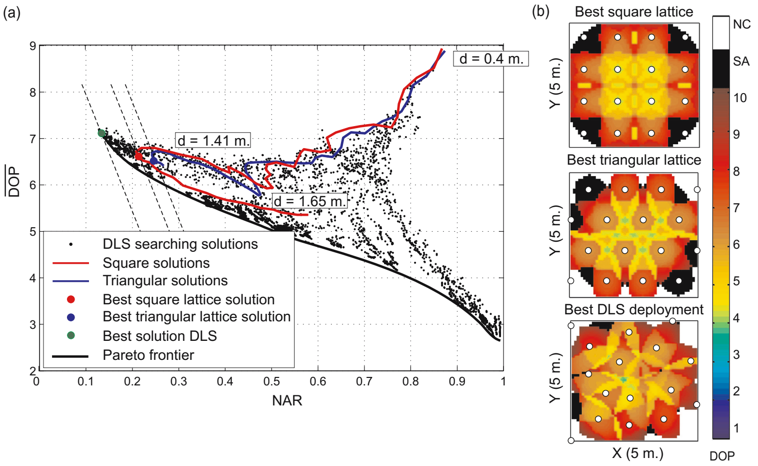

Fig. 5. Optimization results for a square area applying HT: (a) Cost function space, (b) Best sensor deployments

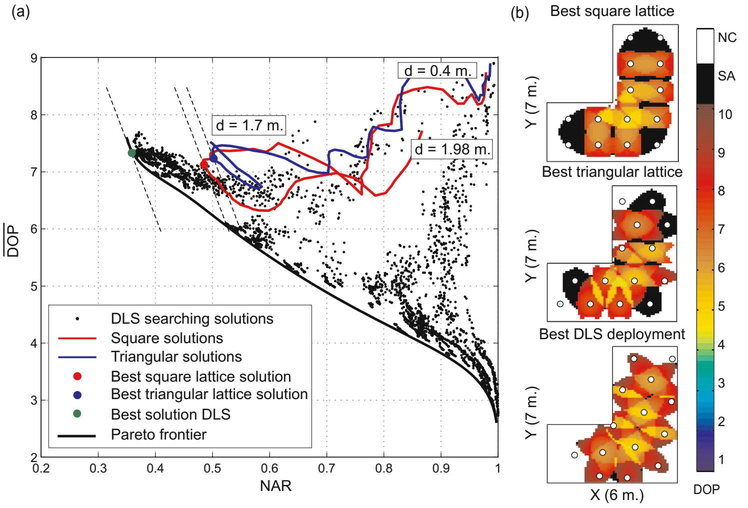

Fig. 6. Optimization results for a L-shaped area applying HT: (a) Cost function space, (b) Best sensor deployments

The performance curves between DOP and NAR for regular lattices and DLS solutions, applying ST, in the square and L-shape area, are presented in the figure 3a and 4a, respectively. The black trace is the *Pareto Frontier*, which was found with DLS method. Red and blue traces represent all of the solutions that were found for square and triangular sensor deployments, changing uniformly inter-sensor distances (d). The green, red and blue points are the best DLS, square and triangular found solutions, respectively. The dashed straight line represents the linear combination of precision and availability using the selected values for K1 and K2 (15 and 400, respectively) in the equation (2). The DLS solution was the best found. However, the best lattices solutions were suboptimal, a good DOP was achieved, but the NAR was poor for square and triangular lattices. The best selected sensor deployments are shown in figure 3b and 4b for square and L-shape area, respectively. White circles, black and white zones represent the sensors, SA and NC, respectively. The colored bar allows to identify the best (blue) and worst (red) DOP areas.

Applying HT method in both areas, the best found sensor deployments based on regular lattices present more NAR than the best DLS solution, keeping a similar DOP values. Therefore, DLS solutions remain as the preferable option. Figure 5 and 6 show the results for the square and L-shaped area, respectively. Figure 5b and 6b show that the best DLS deployments generated less SA than the best regular deployments, where generated SA were considerable. Besides in DLS deployment solution a sensor was exactly located in the out-corner of L-shape area, maximizing the availability. Values of $f(\Omega)$, DOP and NAR, for the best found sensor deployment, are summarized in the table 1.

4 Discussion and Conclusions

Regular sensor distributions generate singular areas and their coverage is poor. These problems appear when the positioning system uses few sensors and become bigger when it apply HT than ST. Optimization methods such as DLS allow that sensor distribution adapts to the shape of the location area. It was notable in the L-shaped area since using DLS the sensor locations become adaptable to the shape of area, avoiding obstacles and achieving more coverage. This adapted distributions are not intuitive. Therefore, these tools are necessary in order to get the best coordinates of sensor locations. Making a comparison between the found DLS *Pareto Frontier* and the curves found with regular deployments, it is observed that the best solutions were always obtained using DLS method, independently of the arbitrary selected values of K1 and K2. If the positioning system has a redundant amount of sensors, a deployment solution based on regular lattices is sufficient since good precisions and availability are achieved. Therefore, DLS solutions are useful only when it is desired to use few sensors in the system.

This work studied how an optimization process as DLS is able to find optimal sensor deployments that outperform regular distributions. This search strategy avoids singularities and achieves better coverage with good positioning precision. These results are important for sensor network deployment with minimum infrastructure costs.

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