A review of multidimensional scaling techniques for RSS-based WSN localization

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Abstract—**Mobile nodes with sensing, computing and communicating capabilities are usually deployed in indoor environments as wireless sensor networks (WSN) for internet-of-things applications. Proper interpretation of the data acquired by these sensor networks requires an estimate of their location. Among the currently existing positioning methods for WSN, multidimensional scaling (MDS) is a computationally efficient technique based on empirical measurement or estimation of the ranges between them. This communication describes and evaluates several versions of MDS localization methods, particularly applied to indoor networks based on transmission of radiofrequency signals and measurement of the received signal strengths (RSS). The positioning accuracy, resistance to measurement noise, and computation times are analyzed and commented within a common framework.**

I. Introduction

Wireless sensor networks (WSN) are composed of multiple small nodes deployed in an exterior or indoor environment with the purpose of measuring some relevant environmental parameters. Usually, the information collected and transmitted by the nodes must be accompanied of an estimate of their physical locations (with respect to an external reference system) to be meaningful; however, manual measurement of each node location during deployment can be impossible or non-practical. Thus, it is important to have some device by which the nodes can exchange signals with their neighbours with the final objective of achieving a location estimate of the complete WSN [1]. Usually, this is achieved with acoustic or radiofrequency signals.

Many methods exist for WSN node localization (see [2], chapters 23-27). Direct least-square minimization of the residues between the measurements and the predicted values normalized by the measurement variance provides the maximum likelihood estimate of node locations, but the process is difficult in practice due to the high dimensionality of the search space. Other popular suboptimal methods include minimization techniques such as linear programming, relaxation or set theoretic techniques, and Bayesian techniques such as belief propagation, which estimates the location of all nodes in a distributed way.

Multidimensional scaling (MDS) is a mathematical method for visualization and classification of high-dimensional data, originally developed in the fields of psychology and psychometrics [3], from which its application scope has extended to other disciplines. MDS receives a matrix with the distances between elements, and produces for each element a vector of coordinates in a Euclidean space of lower dimension. MDS is well suited for cooperative localization problems, where a set of mobile devices have to estimate their relative locations from the measured ranges between themselves (called metric MDS, since the ranges correspond to the Euclidean distance between the nodes). For this reason, the last decade saw a number of works [4], [5], [6], [7], [8], [9], [10], in which different variants of this technique were applied to the location of single and multiple nodes in fully and partially connected networks, centralized and distributed solutions, etc.

A key element of MDS localization is the reconstruction of the so called square distance matrix, containing the measured ranges between each pair of nodes. Since this can rarely be achieved in indoor experimental conditions (due to the limited transmitting range of nodes, existence of obstacles, etc), a method for filling in the missing entries of the square distance matrix must be achieved. Recently, some matrix reconstruction techniques have been applied to this end; we will review the most significant and their relationship to MDS localization.

This article evaluates the performance of several MDS-based algorithms for WSN positioning in indoor spaces, based on the measurement of the received signal strength (RSS) of RF signals exchanged among them. Background theory and a succint description of each method is given first. We begin by introducing the general framework of multidimensional scaling methods, and then determine the strengths and weaknesses of these methods with a simulated indoor network: positioning accuracy, computational effort and converge rate are evaluated and compared. Applicability of MDS/subspace techniques to the case of one single mobile node is also briefly considered.

II. Wireless sensor network localization

A wireless sensor network consists of a set of N nodes with two-dimensional locations $\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ (figure 1). During deployment of the network, the location of the nodes is not registered except for a small fraction (N_a) of them, called *anchor nodes*; the remaining $N_m = N - N_a$ are mobile nodes at unknown locations to be determined. However, all nodes have the possibility to exchange signals with their (close) neighbours and then have an estimate of their relative range. Systems using ultrasonic or ultrawideband radio signals will be

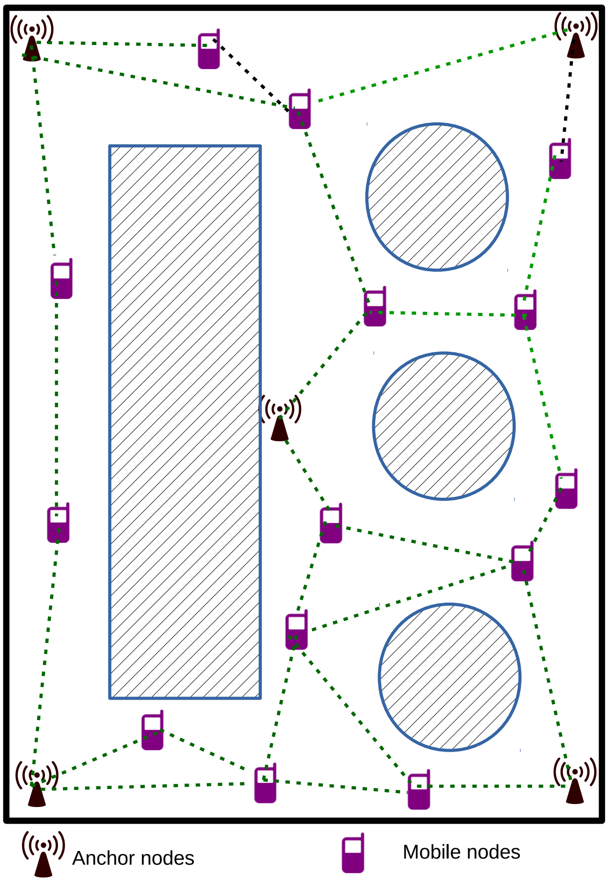

Fig. 1. Localization of the nodes of a wireless sensor network in an indoor environment. Anchor nodes have known positions, while mobile nodes have unknown positions to be determined. Neighboring nodes can measure the received signal strength of the RF signals exchanged between them (dashed lines).

able to directly measure the physical distance between nodes; most common RF-based WSN can only measure the received signal strength (RSS) which does provide only a very rough estimate of the range (particularly in indoor environments). We are mostly concerned with this case in this paper.

We assume that the received signal strength of the RF signal between nodes i and j can be written as $z_{ij} = h(\mathbf{x}_i, \mathbf{x}_j) + e_{ij}$, where z is the measured RSS value, $\mathbf{x}_i, \mathbf{x}_j$ the respective node locations, h a known function, and $e_{ij} = \mathcal{N}(0, \sigma_{ij}^2)$ is Gaussian distributed measurement error with zero mean and variance σ_{ij}^2. The maximum likelihood estimate (MLE) of the positions of the mobile nodes is that which minimizes the cost function:

$$V(\mathbf{x}) = \sum_{i=1}^{N} \sum_{j \in \mathcal{H}(i)} \frac{1}{\sigma_{ij}^2} (z_{ij} - h(\mathbf{x}_i, \mathbf{x}_j))^2, \tag{1}$$

where $\mathcal{H}(i)$ is the set of nodes connected to node i, and no distinction has been made between mobile and anchor nodes. This cost function is nonlinear and nonconvex, and therefore can have many local minima. Maximum likelihood estimates $\widehat{\mathbf{x}}_{\text{MLE}}$ of equation 1 can in principle be obtained by a systematic search (a grid) through all possible locations in the set $\mathbf{x}_m$; however, this approach is computationally expensive due to the high dimensionality ($2N_m$) of the solution space.

We still need a functional way to link RSS readings to the node locations. Determining this model exactly is impossible due to the complex characteristics of RF propagation in indoor environments, so some simplifications have to be made. In this work well-known path loss law [11]:

$$z_{ij} = h(\mathbf{x}_i, \mathbf{x}_j) = \text{RSS}_0 - 10\alpha \log_{10} \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{d_0} + e_{\text{RSS}}, \tag{2}$$

where d_0 is a reference distance, RSS_0 is the signal strength at distance d_0, and α is the path loss exponent. The term e_{RSS} is fading noise corresponding to the unmodeled effects of indoor propagation of RF signals; we will consider that it is Gaussian distributed, $e_{\text{RSS}} \sim \mathcal{N}(0, \sigma_{\text{RSS}}^2)$. Further, for this paper we will assume that parameters RSS_0, α and σ_{RSS} are known or estimated from a few RSS measurements taken at the relevant indoor environment.

III. The Multidimensional Scaling Approach

Assume a WSN network formed by N nodes, with locations unknown, but fully connected; i.e., all RSS_{ij} for $i, j = 1, \ldots, N$ are measured, and corresponding ranges r_{ij} can be estimated. Consider a square distance matrix defined as:

$$\mathbf{D} = \begin{bmatrix} 0 & r_{12}^2 & r_{13}^2 & \ldots & r_{1N}^2 \\ r_{12}^2 & 0 & r_{23}^2 & \ldots & r_{2N}^2 \\ r_{13}^2 & r_{23}^2 & 0 & \ldots & r_{3N}^2 \\ & & \ldots & & \\ r_{1N}^2 & r_{2N}^2 & r_{3N}^2 & \ldots & 0 \end{bmatrix}, \tag{3}$$

where no formal distinction is made between anchors and mobile nodes. Matrix $\mathbf{D}$ has an important property which is exploited in subspace decomposition methods: its rank is at most 4 for two-dimensional localization (5 for three-dimensional localization).

The rank of $\mathbf{D}$ can be further reduced by double centering:

$$\mathbf{B} = -\frac{1}{2}\mathbf{JDJ}, \tag{4}$$

with $\mathbf{J} = 1_N - \frac{1}{N}\mathbf{e}_N\mathbf{e}_N^T$ is a centering matrix, 1_N is the $N \times N$ identity matrix and $\mathbf{e}_N$ is a column vector of length N with all elements equal to one. Then $\mathbf{B} \simeq \mathbf{XX}^T$, where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N]^T$ is a $N \times 2$ matrix with the node coordinates translated such that the centroid of the network is at coordinates $(0, 0)$; the relationship is only approximate because of measurement noise. The matrix containing the relative beacon locations can be estimating by minimizing the quantity:

$$\widehat{\mathbf{X}}_{\text{MDS}} = \arg\min_{\mathbf{X}} \|\mathbf{B} - \mathbf{XX}^T\|^2. \tag{5}$$

To obtain the solution to equation 5, we perform an eigenvalue decomposition of $\mathbf{B}$:

$$\mathbf{B} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T, \tag{6}$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ is the eigenvalue matrix, and $\mathbf{V}$ the corresponding eigenvector matrix. It can be proven that matrix $\mathbf{B}$ is symmetric and positive-definite and has at most rank 2, which implies that it can be represented by its two largest eigenvalues (which span the signal subspace). Then $\mathbf{B} \simeq \mathbf{V_s \Lambda_s V_s}$, and the solution to equation 5 is:

$$\widehat{\mathbf{X}}_{\text{MDS}}^{\text{signal}} = \mathbf{V_s \Lambda_s}^{1/2}, \tag{7}$$

with $\mathbf{\Lambda_s}^{1/2} = \text{diag}(\lambda_{s1}^{1/2}, \lambda_{s2}^{1/2})$ is a diagonal matrix with the two largest eigenvalues. Note that the obtained position estimate $\widehat{\mathbf{X}}_{\text{MDS}}$ is not referred to any particular reference system, and will be in general, translated, rotated and possibly reflected from its physical arrangement. The true location of the mobile nodes can be recovered using the known positions of anchor nodes.

Instead of the signal subspace solution, some MDS methods consider the noise subspace, which is spanned by the remaining autovectors of the eigenvalue factorization: $\mathbf{V}_n = [\mathbf{v}_3 \ldots, \mathbf{v}_N]$. Using the property $\mathbf{V_s V_s}^T + \mathbf{V_n V_n}^T = \mathbf{1}_N$, a second solution can be derived for the position:

$$\widehat{\mathbf{X}}_{\text{MDS}}^{\text{noise}} = \frac{\mathbf{e}_N^T \mathbf{V}_n \mathbf{V}_n^T}{\mathbf{e}_N^T \mathbf{V}_n \mathbf{V}_n^T \mathbf{e}_N} \mathbf{X}_a, \tag{8}$$

where $\mathbf{X}_a$ is a matrix containing the coordinates of the anchor nodes. Unlike the previous solution, this noise subspace estimate does not require translation, rotation or reflection.

The most notable advantages of MDS-based localization are that it provides a closed-form estimate of network localization, regardless of network size; and that the method is resistant to individual error ranges, given the data redundancy present in the square distance matrix. By contrary, MDS methods do not approach CRLB performance, and indeed do not fully exploit all available measurements. This is due to the requirement that the square distance matrix be symmetric; then RSS measurements between two nodes (RSS_{ij} and RSS_{ji}) must be combined to give a single d_{ij}^2, with some information being discarded in the averaging process.

As multidimensional scaling works with distances between nodes, inversion of equation 2 is needed to obtain ranges from RSS values. Note that direct inversion of equation 2:

$$\widehat{d}_{\text{biased}}^2 = d_0^2 \cdot \exp\left(-\frac{\text{RSS}_{ij} - \text{RSS}_0}{5\alpha}\right) \tag{9}$$

leads to a biased estimate. The following estimator [12] for d_{ij}^2 is unbiased:

$$\widehat{d}_{\text{unbiased}}^2 = d_0^2 \cdot \exp\left(-\frac{\text{RSS}_{ij} - \text{RSS}_0}{5\alpha}\right) \cdot \exp\left(-\frac{\ln 10 \sigma_{\text{RSS}}^2}{50\alpha^2}\right). \tag{10}$$

IV. Versions of MDS applied to WSN localization

In real wireless sensor networks (WSN), especially in indoor environments, nodes can only communicate with a small set of neighbouring nodes, due to physical obstacles or limited RF transmission power. Since the network is not fully connected, and the square distance matrix of equation 3 has many blank entries, direct estimation of node locations via MDS is not feasible.

For the purposes of this paper, MDS methods are classified into four broad categories:

1) General reconstruction methods for the square distance matrix that do not take into account its algebraic properties. Among them the MDS-MAP variants which use generic short-distance algorithms to estimate the missing ranges between nodes.
2) Reconstruction methods which use properties of the square distance matrix to fill for its blank entries: among them SVD-reconstruct and Iterative Inertial approximation.
3) Iterative minimization methods which operate directly on the incomplete square distance matrix, like the stress majorization of a convex function (SMACOF).
4) Projection methods such as Fastmap.

These methods will be described in the following subsections, and their performance compared to two non-MDS localization algorithms: the push-pull estimation (PPE) algorithm, and a naive method by direct optimiztion of the MLE cost function.

A. General reconstruction methods for the square distance matrix

In the **MDS-MAP method** [4], the missing entries of the square distance matrix are reconstructed with shortest-path algorithms for graphs such as Dijkstra's or Floyd-Warshall's method. Then the MDS method uses this approximate distance matrix as explained in section III for the signal [4] or noise [13] subspaces. Both solutions are suboptimal unless the measurement covariance matrix is included and residues from the eigenvalue decomposition are reweighted. However, WMDS can achieve CRLB performance in the case that all RSS measurements between nodes are available, and noise levels are low. One big drawback of **weighted MDS** [14] is that it requires a few iterations of the residue reweighting step in which inversion of a $N(N - N_a) \times N(N - N_a)$ matrix is carried out. This is very time consuming for medium to large networks.

B. Reconstruction methods for the square distance matrix using its low-rank properties

These MDS techniques are based on the known redundancy of low-rank matrices such as the square distance matrix.

SVD-Reconstruct is based on sampling algorithms in which only a limited number of entries of the square distance matrix can be determined (sampled) [15]. The method relies on the property that the rank of the square distance matrix is at most 4 (for two dimensional localization). So, in principle, knowledge of only $8N$ entries is needed to reconstruct without error the complete square distance matrix. The procedure suggested consists in building a matrix $\mathbf{S}$ with the following entries:

$$S_{ij} = \begin{cases} \frac{r_{ij}^2 - \gamma_{ij}(1-p_{ij})}{p_{ij}} & \text{if } r_{ij} \text{ was measured} \\ \gamma_{ij} & \text{otherwise,} \end{cases} \tag{11}$$

where p_{ij} is an estimate of the detection probability of measurement r_{ij}^2, and γ_{ij} is an estimate of r_{ij}^2 in the case that it was not measured. Suitable values for probability p_{ij} and unmeasured range γ_{ij} can be estimated from a model or from experimental measurements. Alternatively, a simple detection region model (step function) can be used, with $p_{ij} = 1$ if $d_{ij} \leq R_{\text{det}}$ and 0 otherwise, where R_{det} is a typical detection range.

The next step is to build $\mathbf{S}_4$ being the rank 4 approximation to $\mathbf{S}$, obtained by singular value decomposition. It is proved in [15] that the Frobenius norm of $\|\mathbf{D} - \mathbf{S}_4\|_F$ is bounded. Since $\mathbf{S}_4$ is a good approximation of the square distance matrix, the MDS procedure is performed on it, instead of using the incomplete matrix $\mathbf{D}$.

With the use of the PLL model (equation 2), estimates for p_{ij} can be produced, provided we know the values of the parameters of the PLL equation, and the detection threshold RSS_{th}. In an environment without obstacles, the detection probability is given by [16]:

$$p_{ij} = Q\left(\frac{\text{RSS}_{\text{th}} - \text{RSS}(d_{ij})}{\sigma}\right), \tag{12}$$

with $Q(t) = 1/\sqrt{2\pi}\int_t^\infty e^{-x^2/2}\,dx$ being the complementary error function. For an indoor environment with obstacles, p_{ij} can be precomputed by numerical methods.

In [17], a different property of the square distance matrix is employed: its inertia, defined as the number of positive and negative eigenvalues: (ρ_+, ρ_-). Given that $\mathbf{D}$ is symmetric and of rank 4, it has ideally one positive and three negative eigenvalues, i.e., its inertia is $(1, 3)$. Using this property, if we eigen-decompose any estimate $\widehat{\mathbf{D}}$ of the true square distance matrix as $\widehat{\mathbf{D}} = \mathbf{V}\mathbf{\Lambda}\mathbf{D}^T$, then the inertial $(1, 3)$ approximation is given as:

$$\mathbf{E} = \lambda_+\mathbf{v}_+\mathbf{v}_+^T + \lambda_{1-}\mathbf{v}_{1-}\mathbf{v}_{1-}^T + \lambda_{2-}\mathbf{v}_{2-}\mathbf{v}_{2-}^T + \lambda_{3-}\mathbf{v}_{3-}\mathbf{v}_{3-}^T, \tag{13}$$

with λ_+ and λ_- being the positive and negative eigenvalues respectively, and $\mathbf{v}_{i+}$, $\mathbf{v}_{i-}$ the corresponding eigenvectors. An approximation $\widehat{\mathbf{D}}$ of the square distance matrix $\mathbf{D}$ is built iteratively as follows:

1) Build $\widehat{\mathbf{D}}_0$ as $\mathbf{D}$ with unknown entries being replaced by γ_{ij}.
2) For $k = 0, 1, \ldots$:
3) Compute $\mathbf{E}_k$ by SVD decomposition of $\widehat{\mathbf{D}}_k$ as per equation 13.
4) Create $\widehat{\mathbf{D}}_{k+1}$, with the ij-th element given by r_{ij}^2 if measured, or by the ij-th element of $\widehat{\mathbf{E}}_k$ if unknown.
5) Iterate until convergence.

C. SMACOF

Another possibility is the **SMACOF method** (Scaling by majorizing a convex function), which consists in direct minimization of a stress function $\sigma(\mathbf{X})$ given as [5]:

$$\sigma(\mathbf{X}) = \sum_{i=1}^{N}\sum_{j>i}^{N} w_{ij}(\widehat{r}_{ij}(\mathbf{X}) - r_{ij})^2, \tag{14}$$

which measures the discrepancy between the ranges computed from currently estimated beacon distribution, $\widehat{r}_{ij}(\mathbf{X})$, and those experimentally measured, r_{ij}, with $w_{ij} = 1/\sigma_{ij}^2$ if nodes i and j are connected, and 0 if they are not. This method deals with the original $\mathbf{D}$ matrix without attempting to reconstruct the missing values. The solution $\mathbf{X}$ which minimizes the stress can be found iteratively in the following way.

1) Initialize the node positions $\mathbf{X}_0$ (anchor nodes are assigned to their known positions).
2) For each iteration (k), compute the $N \times N$ matrices $\mathbf{B}$ and $\mathbf{V}$:
$$\begin{aligned} B &= \textstyle\sum_{i=1}^{N}\sum_{j>i}^{N} \frac{w_{ij}r_{ij}}{\widehat{r}_{ij}(\mathbf{X}_k)} A(i,j) \\ V &= \textstyle\sum_{i=1}^{N}\sum_{j>i}^{N} w_{ij}A(i,j) \end{aligned} \tag{15}$$
where for each pair (i, j), $\mathbf{A}(i, j)$ is an $N \times N$ auxiliary matrix with only four nonzero elements: $a_{ii} = a_{jj} = 1, a_{ij} = a_{ji} = -1$. A new position estimate is computed as
$$\mathbf{X}_{k+1} = \mathbf{V}^{-1}\mathbf{B}\mathbf{X}_k,$$
with $\mathbf{V}^{-1}$ being the pseudoinverse of matrix $\mathbf{V}$.
3) Proceed while condition $\sigma(\mathbf{X}_{k+1}) < \sigma(\mathbf{X}_k)$ is fulfilled.

Upon convergence of the SMACOF algorithm, the function being minimized is actually $\sum_{i,j} w_{ij}(\widehat{r}_{ij}(\mathbf{X})^2 - r_{ij}^2)^2$, so, while CRLB accuracy might not be reached, for practical purposes the SMACOF solution is usually close enough to the true solution.

D. Fastmap

Fastmap [8] is a dimensionality reduction technique which projects the coordinates of the mobile nodes into lines connecting a set of anchor nodes (called pivots). For example, if unknown position node i has access to anchor nodes a, b and c, its first coordinate on the axis connecting $\mathbf{x}_a$ and $\mathbf{x}_b$ is given by the cosine law:

$$x_i' = \frac{r_{ai}^2 - r_{bi}^2 + d_{ab}^2}{2d_{ab}},$$

while the second coordinate can be projected on the line connecting $\mathbf{x}_a$ and $\mathbf{x}_c$:

$$y_i' = \frac{\tilde{r}_{ai}^2 - \tilde{r}_{ci}^2 + d_{ac}^2}{2d_{ac}},$$

with $\tilde{r}_{ai}^2 = r_{ai}^2 - {x'}_i^2$ and $\tilde{r}_{bi}^2 = r_{bi}^2 - {x'}_i^2$. The real position of the node can be estimated from its (x_i', y_i') coordinates and the known coordinates of the pivot nodes. This procedure can be performed sequentially with all nodes, using as new pivots mobile nodes whose position has already been determined. At each step, nodes with the largest norm of the cross product $\|\vec{ab} \times \vec{ac}\|$ should be chosen as pivots. As the positions of new nodes are estimated solely from the pivot nodes (i.e., there is no averaging), Fastmap is sensitive to coordinate alignment, and usually provides less accurate results, especially for sparse networks.

E. Non-MDS methods

For performance comparison, we include two methods for WSN location estimation not based on multidimensional scaling, but rather on minimization of the ML cost function. These methods can potentially converge to the ML solution, but they are slow, and subject to local minima solutions.

- A push-pull estimate (PPE) algorithm [18]. This is a spring method which envisions mobile nodes being affected by a "force" which is proportional to the difference between the measured and the expected RSS values from the neigbouring nodes. The force experimented by node i from node j is given by: $\mathbf{f}_{ij} = (\text{RSS}_{ij}(\mathbf{x}) - \text{RSS}_{ij})\mathbf{e}_{ij}$, where $\mathbf{e}_{ij}$ is the unit vector from $\mathbf{x}_j$ to $\mathbf{x}_i$. The sum of all forces acting on node i is then $\mathbf{F}_i = 1/n_i \sum_j \mathbf{f}_{ij}$. Then the node locations move iteratively as: $\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \alpha \mathbf{F}_i$, where α is a scalar value affecting the movement "speed" of the nodes (equivalent to the interval of the gradient descent method).
- Direct minimization of the cost function (equation 1). The nodes are displaced one at a time from their current locations by small amounts, measuring whether this leads to a decrease of the cost function.

In both cases, mobile nodes are initialized to the centroid of the anchor nodes they are connected to, or to the centroid of all anchor nodes if they are not connected to any.

V. Evaluation of performance of algorithms

This section presents an evaluation of the performance of the MDS methods described in section IV. As a testbed for localization of anchor and mobile nodes we use the indoor area of figure 2, which consists in a 90×100 m area, with three obstacles that block completely or partially the RF transmissions. The network consists of 7 anchors and 69 unknown position nodes (arranged in this case in a rectangular grid for easier visualization of results). The parameters of the PLL are $\text{RSS}_0 = -50$ dBm, $\alpha = 2.5$ dBm, $r_0 = 1$ m, $\sigma_{\text{RSS}} = 10$ dBm, and $\text{RSS}_{th} = -100$ dBm, and are supposed to be known to the MDS algorithms. In this way, an estimate of the measurement covariance matrix can be computed from the measured RSS values themselves.

A. Accuracy of methods

In the evaluation, the MDS-based methods of section IV are compared with an iterative method (PPE or push-pull estimation) [18], which is a form of gradient descent. This method can potentially reach the maximum likelihood estimate of the true location provided that it does not get stuck in a local minimum. The mobile nodes are initiated either at the centroid of the anchor locations, or at the corresponding MDS-MAP estimates. For iterative methods which need a terminating condition, we stop the loops when each node's displacement between iterations falls below $dx = 1$ cm.

Figure 3 shows the network positioning results for each method. It is seen that, among the MDS methods, MDS-MAP, weighted MDS-MAP and SMACOF most MDS versions

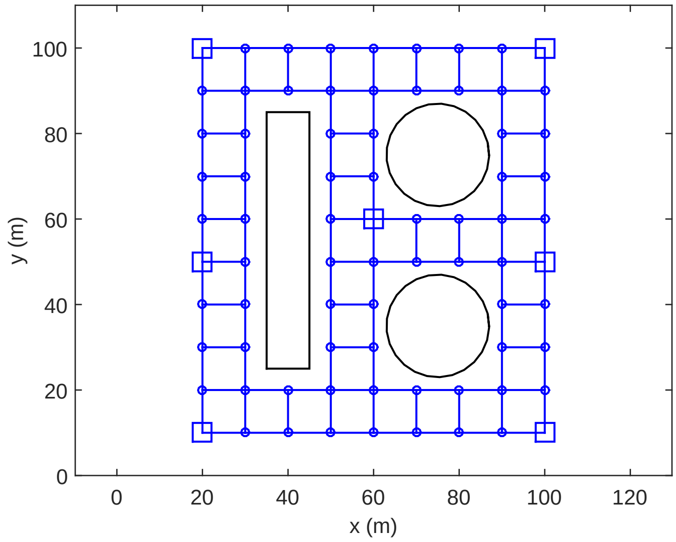

Fig. 2. Indoor testbed for evaluation of the MDS positioning methods. Nodes are arranged in a rectangular grid of step 10 m for this example. Square marks are anchor nodes with known location, circular marks are mobile nodes with unknown location. One rectangular and two circular obstacles impede node communication throughout them.

manage to recreate faithfully the shape of the network, while the iterative inertial approximation only gives a rough idea of the node distribution, and SVD reconstruct and Fastmap fare poorly. The last two MLE-estimate methods obtain good results. Note that the MDS-MAP versions show some deformity in the left part of the network, due to the way that the incomplete links are reconstructed (according to topological rather than metric approximations); this is not observed in SMACOF since this algorithm does not attempt to reconstruct missing links. Fastmap does not handle well the imprecise ranges estimated from the noisy RSS measurements, and the reconstructed network is not recognizable. Of the two low-rank reconstruction algorithms, iterative inertial approximation is more robust, and SVD reconstruct does not result in a sensible solution. Our feeling is that methods based on low rank matrix properties are only effective for matrices with only a few blank entries, which is not practical in indoor localization.

The corresponding CDFs for the node positioning errors are shown in figure 4. It is seen that only SMACOF approaches the behaviour of MLE estimates.

B. Dependence on network connectivity

In this experiment we evaluate the performance of the methods with respect to network connectivity, defined as the average number of nodes which are connected to any node. For this purpose, we change the RSS detection threshold (RSS_{th}) and signal blocking factor by room obstacles. This permits to control the number of active links in the complete network.

Results are shown in figure 5. Again, of the MDS-based techniques, the best behaviour is shown by SMACOF, which achieves performance close to MLE methods, while MDS-MAP obtains an error approximately constant with network

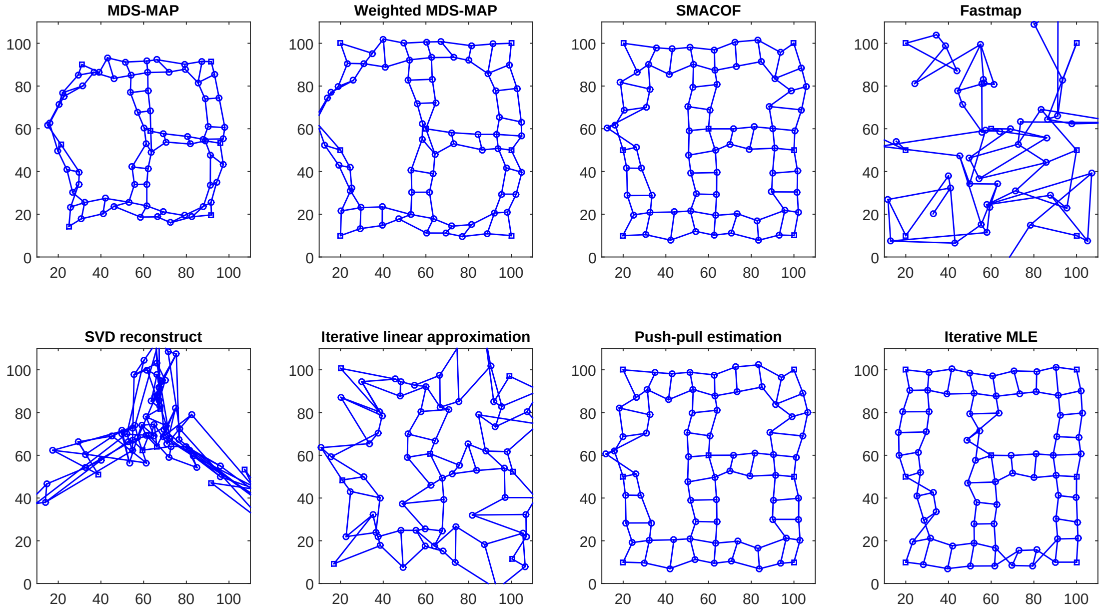

Fig. 3. Performance of the MDS methods for the conditions of section V.

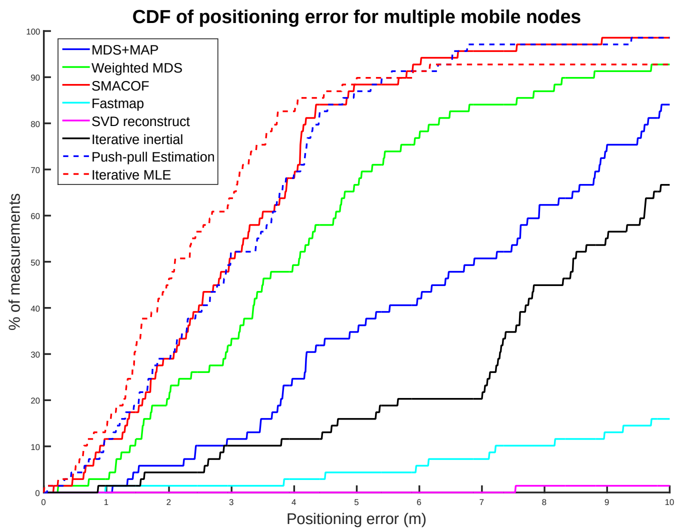

Fig. 4. Cumulative distribution function (CDF) of the positioning error of the estimated sensor arrangements of figure 3.

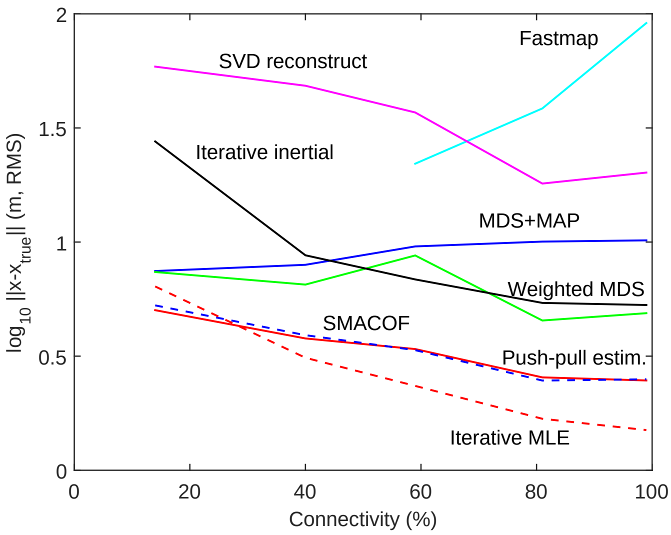

Fig. 5. Dependence of the RMS positioning error of the network, with the network connectivity, obtained with an average of 10 simulations.

connectivity. Weighted MDS and the iterative linear approximation show intermediate performance.

C. Dependence on noise amplitude

In this experiment we evaluate the performance of the methods with respect to noise amplitude, given by parameter σ_{RSS}. Results are shown in figure 6. Again, SMACOF is the more robust method against noise, achieving performance close to the MLE iterative methods. The iterative inertial approximation and the weighted MDS are marginally better than plain MDS-MAP, although their computational cost is much higher. Fastmap and SVD-reconstruct give, in general, poor performance.

D. Computational load

This section briefly analyzes the computational load of the MDS methods, and compares it with non MDS-based techniques. We have not performed a count of operations for each method, but instead relied on Matlab's execution time for each; some care has been put to optimize the code for each

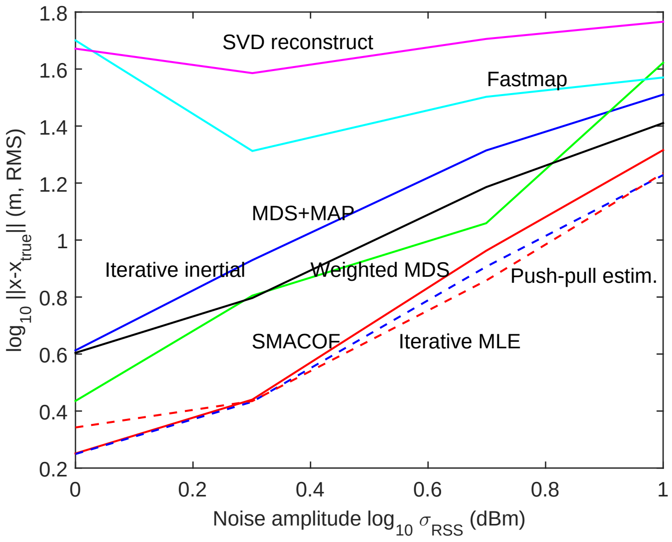

Fig. 6. Dependence of the RMS positioning error of the network, with the noise amplitude, obtained with an average of 10 simulations.

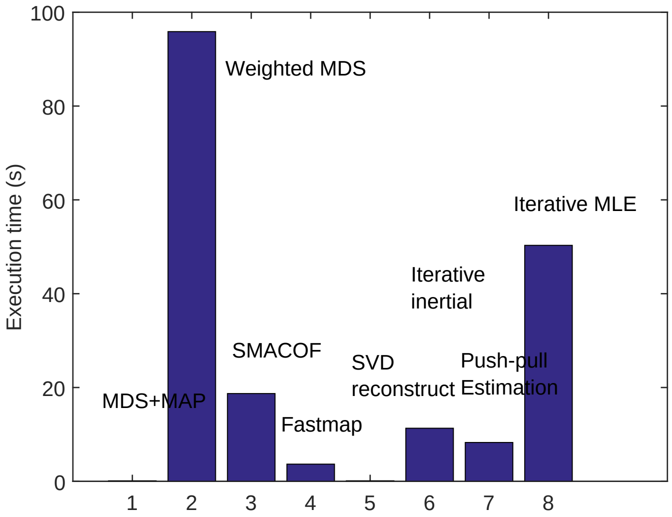

Fig. 7. Execution time of the MDS locatization techniques of section IV as evaluated with Matlab.

method. The results are shown in figure 7.

By far the most demanding method is weighted MDS. Although the method converges in a few iterations, it requires the inversion of a large reweighting matrix (see section IV), which is computationally demanding. The sample network of figure 2, with 7 anchor and 69 mobile nodes, requires the inversion of a matrix of size 5244×5244, which takes about one and a half minute in Matlab. Comparatively, the SMACOF method needs many more iterations, but takes up less processing time because it handles smaller size matrices. The iterative MLE methods (PPE and cost function minimization) achieve close to CRLB performance, and require, in our testbed, the second longest execution time. MDS-MAP executes very fast but has some imprecise results.

E. MDS applied to a single mobile node

As a particular case, some authors have applied multidimensional scaling methods to position estimation of a single node (taking $N = N_a + 1$). In a **direct implementation of MDS**, the computed network is translated and rotated from the solution excluding the mobile node. This is not optimal because the condition that the centroid of the network lies at the origin (implicit in the double centering equation 4) cannot be fulfilled exactly. This shortcoming is compensated by a **modification of MDS** [6] in which the mobile node's position is excluded from the centroid computation. The work [7] proposes to use the **noise subspace** in equation 8 instead of the signal subspace of equation 6. A later work [10] showed that these three MDS variants are actually equivalent, varying only in their choice of centroid for the network. Still, one weakness common to the standard MDS methods is that the measurement range noise is not incorporated to the solution. The **weighted MDS** method in [9] reweights the residues of the subspace decomposition, permitting to incorporate the measurement covariance error and improve the position estimates. Finally, among the most recent works, we can mention the method in [19], which employs a subspace decomposition of the square distance matrix D based upon **Lagrange multipliers** instead of eigenvalue decomposition.

As part of this work, we evaluated these methods in a similar indoor setup, considering localization of the single mobile node when placed inside or outside of the are delimited by the anchor nodes. The evaluation rendered the following results: the direct and modified versions of MDS showed very similar performance, with the noise subspace MDS improving slightly upon them (only for locations inside of the area delimited by the anchor nodes), and the Lagrange multipliers MDS being clearly inferior. A significant improvement is obtained by the weighted MDS method, which considers the measurement error covariance matrix. This weighted MDS method, although more complex to implement, achieves optimal performance (the Cramér-Rao lower bound limit). Note however that Taylor minimization performs as good as weighted MDS, and is simpler to apply. The conclusion is that there is no significant advantage to use MDS methods for localization of a single node over the Taylor minimization method, a result that can be applied to many approximate "closed-form" solutions to the positioning problem [20].

VI. Conclusions

In this communication we have reviewed the theory and provided a comparison by simulation of several variants of the multidimensional scaling (MDS) method used for positioning of wireless sensor networks. In indoor environments, this results in a sparse square distance matrix, since most nodes will not be within communication range. We have evaluated several MDS methods with typical received signal strength measurements. This results in higher range variances than those commonly found in time-of-flight systems.

We have compared several MDS versions and compared them with two maximum likelihood techniques: a push-pull estimate algorithm, and direct minimization of the cost function in equation 1.

The simplest method is MDS-MAP, which is plain MDS with the missing ranges computed by a generic shortest-path computation technique (such as Dijkstra or Floyd-Warshall). More precise results are obtained by weighting the residues, but the neccesary matricial operations are unwieldy even for medium-sized networks. The iterative algorithm SMACOF provides accurate results and works with low or high connectivity networks. The matrix completion methods (SVD-Reconstruct and Iterative Inertial Approximation) do not perform satisfactorily unless the network is deeply connected, which is not a practical circumstance. Fastmap also provides poor results.

For this reason, the most convenient MDS method in our experience is SMACOF, an iterative minimization technique with relatively small computational load which performs close to the CRLB after connectivity reaches a threshold value.

MDS methods have been used as an initialization method to MLE position estimation methods [21] or to Bayesian tracking of people in indoor environments [22]. Given the power and simplicity of MDS methods, it would be interesting to extend research in this area to the tracking of dynamic networks, by combining the MDS formulation with Bayesian filters [23], further increasing the applicability of multidimensional scaling techniques for localization and tracking.

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REFERENCES

[1] A. K. Paul and T. Sato, “Localization in wireless sensor networks: A survey on algorithms, measurement techniques, applications and challenges,” *Journal of Sensor and Actuator Networks*, vol. 6, no. 4, p. 24, 2017.

[2] R. Zekavat and R. M. Buehrer, *Handbook of position location: Theory, practice and advances*. John Wiley & Sons, 2011.

[3] S. L. France and J. D. Carroll, “Two-way multidimensional scaling: A review,” *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 41, no. 5, pp. 644–661, 2011.

[4] Y. Shang, W. Ruml, Y. Zhang, and M. P. Fromherz, “Localization from mere connectivity,” in *Proceedings of the 4th ACM international symposium on Mobile ad hoc networking & computing*, 2003, pp. 201–212.

[5] X. Ji and H. Zha, “Sensor positioning in wireless ad-hoc sensor networks using multidimensional scaling,” in *23rd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, vol. 4, 2004, pp. 2652–2661.

[6] K. W. Cheung and H.-C. So, “A multidimensional scaling framework for mobile location using time-of-arrival measurements,” *IEEE Transactions on Signal Processing*, vol. 53, no. 2, pp. 460–470, 2005.

[7] H. So and F. Chan, “A generalized subspace approach for mobile positioning with time-of-arrival measurements,” *IEEE Transactions on Signal Processing*, vol. 55, no. 10, pp. 5103–5107, Oct. 2007.

[8] G. Latsoudas and N. D. Sidiropoulos, “A fast and effective multidimensional scaling approach for node localization in wireless sensor networks,” *IEEE Transactions on Signal Processing*, vol. 55, no. 10, pp. 5121–5127, 2007.

[9] H.-W. Wei, Q. Wan, Z.-X. Chen, and S.-F. Ye, “A novel weighted multidimensional scaling analysis for time-of-arrival-based mobile location,” *Signal Processing, IEEE Transactions on*, vol. 56, no. 7, pp. 3018–3022, July 2008.

[10] Z.-X. Chen, H.-W. Wei, Q. Wan, S.-F. Ye, and W.-L. Yang, “A supplement to multidimensional scaling framework for mobile location: A unified view,” *IEEE Transactions on Signal Processing*, vol. 57, no. 5, pp. 2030–2034, 2009.

[11] S. Y. Seidel and T. S. Rappaport, “914 MHz path loss prediction models for indoor wireless communications in multifloored buildings,” *IEEE Trans. on Antennas and Propagation*, vol. 40, no. 2, pp. 207–217, 1992.

[12] L. Lin, H.-C. So, and F. K. Chan, “Multidimensional scaling approach for node localization using received signal strength measurements,” *Digital Signal Processing*, vol. 34, pp. 39–47, 2014.

[13] F. K. W. Chan, H. C. So, and W. K. Ma, “A novel subspace approach for cooperative localization in wireless sensor networks using range measurements,” *IEEE Transactions on Signal Processing*, vol. 57, no. 1, pp. 260–269, 2009.

[14] F. Chan and H. C. So, “Efficient weighted multidimensional scaling for wireless sensor network localization,” *IEEE Transactions on Signal Processing*, vol. 57, no. 11, pp. 4548–4553, 2009.

[15] P. Drineas, A. Javed, M. Magdon-Ismail, G. Pandurangant, R. Virrankoski, and A. Savvides, “Distance matrix reconstruction from incomplete distance information for sensor network localization,” in *Sensor and Ad Hoc Communications and Networks, 2006. SECON'06. 2006 3rd Annual IEEE Communications Society on*, vol. 2. IEEE, 2006, pp. 536–544.

[16] M. Mansour and D. Waters, “Uncensored indoor positioning,” *IEEE Signal Processing Letters*, vol. 21, no. 7, pp. 824–828, July 2014.

[17] P. A. Regalia and J. Wang, “On distance reconstruction for sensor network localization,” in *Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on*. IEEE, 2010, pp. 2866–2869.

[18] V.-H. Dang, V.-D. Le, Y.-K. Lee, and S. Lee, “Distributed push–pull estimation for node localization in wireless sensor networks,” *Journal of parallel and distributed computing*, vol. 71, no. 3, pp. 471–484, 2011.

[19] S. Qin, Q. Wan, and Z.-X. Chen, “Fast subspace approach for mobile positioning with time-of-arrival measurements,” *IET communications*, vol. 5, no. 14, pp. 2035–2039, 2011.

[20] N. Sirola, “Closed-form algorithms in mobile positioning: Myths and misconceptions,” in *2010 7th Workshop on Positioning Navigation and Communication (WPNC)*, march 2010, pp. 38–44.

[21] X. Li, “Collaborative localization with received-signal strength in wireless sensor networks,” *IEEE Transactions on Vehicular Technology*, vol. 56, no. 6, pp. 3807–3817, 2007.

[22] F. Seco and A. R. Jimnez, “Smartphone-based cooperative indoor localization with rfid technology,” *Sensors*, vol. 18, no. 1, 2018.

[23] W. Cui, C. Wu, W. Meng, B. Li, Y. Zhang, and L. Xie, “Dynamic multidimensional scaling algorithm for 3-d mobile localization,” *IEEE Transactions on Instrumentation and Measurement*, vol. 65, no. 12, pp. 2853–2865, 2016.